dyn_of_lev.mws Equations based on "Dynamics of the Levitron" Roger F Gans et al (J. Phys. D: Appl.Phys.31 1998 671-679)

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with a test of a second order Runge-Kutta integration method and C code generation

6 dec 2003


```
> restart;
> PDEtools[declare]( (q1,q2,q3,q4,q5,q6,p1,p2,p3,p4,p5,p6)(t),
  prime=t, quiet );
> Phi[1]:=Z/(1+Z^2)^(3/2)-3/4*(2*Z^2-3)*Z/(1+Z^2)^(7/2)*(X^2+Y^2):
> Phi:=subs({X=q1,Y=q2,Z=q3},Phi[1]);
```

$$\Phi := \frac{q3}{(1+q3^2)^{(3/2)}} - \frac{3\,(2\,q3^2-3)\,q3\,(q1^2+q2^2)}{4\,(1+q3^2)^{(7/2)}}$$

```
> H:=1/2*(p1^2+p2^2+p3^2+p4^2/a+(p5-p6*cos(q4))^2/(a*sin(q4)^2)+p6
  ^2/c)
  -M*(sin(q4)*(cos(q5)*diff(Phi,q1)+sin(q5)*diff(Phi,q2))+cos(q4)*
  diff(Phi,q3))
  +q3;
```

$$H := \frac{p1^2}{2}+\frac{p2^2}{2}+\frac{p3^2}{2}+\frac{p4^2}{2\,a}+\frac{1}{2}\frac{(p5-p6\cos(q4))^2}{a\sin(q4)^2}+\frac{p6^2}{2\,c}-M\left(\right.$$
$$\sin(q4)\left(-\frac{3}{2}\frac{\cos(q5)\,(2\,q3^2-3)\,q3\,q1}{(1+q3^2)^{(7/2)}}-\frac{3}{2}\frac{\sin(q5)\,(2\,q3^2-3)\,q3\,q2}{(1+q3^2)^{(7/2)}}\right)+\cos(q4)\left(\right.$$
$$\frac{1}{(1+q3^2)^{(3/2)}}-\frac{3\,q3^2}{(1+q3^2)^{(5/2)}}-\frac{3\,q3^2\,(q1^2+q2^2)}{(1+q3^2)^{(7/2)}}-\frac{3\,(2\,q3^2-3)\,(q1^2+q2^2)}{4\,(1+q3^2)^{(7/2)}}$$
$$\left.\left.+\frac{21\,(2\,q3^2-3)\,q3^2\,(q1^2+q2^2)}{4\,(1+q3^2)^{(9/2)}}\right)\right)+q3$$

```
> eqgen:={p1=q[7],p2=q[8],p3=q[9],p4=q[10],p5=q[11],p6=q[12],
          q1=q[1],q2=q[2],q3=q[3],q4=q[ 4],q5=q[ 5],q6=q[ 6]}:
> CodeGeneration[C](subs(eqgen,[dq[1]=diff(H,p1),dq[2]=diff(H,p2),
  dq[3]=diff(H,p3),dq[4]=diff(H,p4),dq[5]=diff(H,p5),dq[6]=diff(H,
  p6),

  dq[7]=diff(-H,q1),dq[8]=diff(-H,q2),dq[9]=diff(-H,q3),dq[10]=dif
  f(-H,q4),dq[11]=diff(-H,q5),dq[12]=diff(-H,q6)]),
          optimize=true);
dq[0] = q[6];
dq[1] = q[7];
dq[2] = q[8];
```

```
t2 = 0.10e1 / a;
dq[3] = q[9] * t2;
t4 = q[11];
t5 = q[3];
t6 = cos(t5);
t8 = q[10] - t4 * t6;
t9 = t8 * t2;
t10 = sin(t5);
t11 = t10 * t10;
t12 = 0.10e1 / t11;
dq[4] = t9 * t12;
dq[5] = -t9 * t12 * t6 + t4 / c;
t17 = q[4];
t18 = cos(t17);
t20 = q[2];
t21 = t20 * t20;
t23 = 0.2e1 * t21 - 0.3e1;
t25 = 0.1e1 + t21;
t26 = t25 * t25;
t28 = sqrt(t25);
t30 = 0.10e1 / t28 / t26 / t25;
t31 = t23 * t20 * t30;
t34 = t21 * t30;
t35 = q[0];
t38 = t23 * t30;
t41 = t23 * t21;
t42 = t26 * t26;
t44 = 0.10e1 / t28 / t42;
dq[6] = M * (-0.3e1 / 0.2e1 * t10 * t18 * t31 + t6 * (-0.6e1 * t34 * t35 - 0.3e
1 / 0.2e1 * t38 * t35 + 0.21e2 / 0.2e1 * t41 * t44 * t35));
t51 = sin(t17);
t55 = q[1];
dq[7] = M * (-0.3e1 / 0.2e1 * t10 * t51 * t31 + t6 * (-0.6e1 * t34 * t55 - 0.3e
1 / 0.2e1 * t38 * t55 + 0.21e2 / 0.2e1 * t41 * t44 * t55));
t67 = t30 * t35;
t70 = t18 * t23;
t73 = t21 * t44;
t78 = t30 * t55;
t81 = t51 * t23;
t90 = 0.10e1 / t28 / t26;
t93 = t21 * t20;
t96 = t20 * t30;
t97 = t35 * t35;
t98 = t55 * t55;
t99 = t97 + t98;
dq[8] = M * (t10 * (-0.6e1 * t18 * t21 * t67 - 0.3e1 / 0.2e1 * t70 * t67 + 0.21
e2 / 0.2e1 * t70 * t73 * t35 - 0.6e1 * t51 * t21 * t78 - 0.3e1 / 0.2e1 * t81 *
t78 + 0.21e2 / 0.2e1 * t81 * t73 * t55) + t6 * (-0.9e1 * t90 * t20 + 0.15e2 * t
93 * t30 - 0.9e1 * t96 * t99 + 0.42e2 * t93 * t44 * t99 + 0.63e2 / 0.4e1 * t23
* t44 * t99 * t20 - 0.189e3 / 0.4e1 * t23 * t93 / t28 / t42 / t25 * t99)) - 0.1
e1;
t123 = t8 * t8;
t129 = t96 * t35;
t131 = t96 * t55;
dq[9] = -t9 / t10 * t4 + t123 * t2 / t11 / t10 * t6 + M * (t6 * (-0.3e1 / 0.2e1
* t70 * t129 - 0.3e1 / 0.2e1 * t81 * t131) - t10 * (0.10e1 / t28 / t25 - 0.3e1
* t21 * t90 - 0.3e1 * t34 * t99 - 0.3e1 / 0.4e1 * t38 * t99 + 0.21e2 / 0.4e1 *
t41 * t44 * t99));
dq[10] = M * t10 * (0.3e1 / 0.2e1 * t81 * t129 - 0.3e1 / 0.2e1 * t70 * t131);
dq[11] = 0.0e0;
>
> pqt:={p1=p1(t),p2=p2(t),p3=p3(t),p4=p4(t),p5=p5(t),p6=p6(t),
```

```
  q1=q1(t),q2=q2(t),q3=q3(t),q4=q4(t),q5=q5(t),q6=q6(t)};
```

$$pqt := \{q1 = q1, q2 = q2, q3 = q3, q4 = q4, q5 = q5, q6 = q6, p1 = p1, p3 = p3, p4 = p4, p5 = p5, p6 = p6, p2 = p2\}$$

```
> eqdp:={diff(p1(t),t)=subs(pqt,diff(-H,q1)),diff(p2(t),t)=subs(pq
  t,diff(-H,q2)),diff(p3(t),t)=subs(pqt,diff(-H,q3)),

  diff(p4(t),t)=subs(pqt,diff(-H,q4)),diff(p5(t),t)=subs(pqt,diff(
  -H,q5)),diff(p6(t),t)=subs(pqt,diff(-H,q6))}:
> eqdq:={diff(q1(t),t)=subs(pqt,diff(H,p1)),diff(q2(t),t)=subs(pqt
  ,diff(H,p2)),diff(q3(t),t)=subs(pqt,diff(H,p3)),

  diff(q4(t),t)=subs(pqt,diff(H,p4)),diff(q5(t),t)=subs(pqt,diff(H
  ,p5)),diff(q6(t),t)=subs(pqt,diff(H,p6))}:
> eq0:={p1(0)=0,p2(0)=0,p3(0)=0,p4(0)=0,p5(0)=Omega*(a*sin(alpha)^
  2+c*cos(alpha)^2)+c*omega*cos(alpha),p6(0)=c*(omega+Omega*cos(al
  pha)),
        q1(0)=0,q2(0)=0,q3(0)=h,q4(0)=alpha,q5(0)=0,q6(0)=0}:
> omega[T]:=7.5;Omega:=1.2;alpha:=0.005;omega:=omega[T]-Omega*cos(
  alpha);a:=0.089;c:=0.139;M:=8.2;h:=1.72;
```

$$\omega_T := 7.5$$

$$\Omega := 1.2$$

$$\alpha := 0.005$$

$$\omega := 6.300015000$$

$$a := 0.089$$

$$c := 0.139$$

$$M := 8.2$$

$$h := 1.72$$

```
> evalf(eq0);
```

$$\{p5(0) = 1.042489639, q3(0) = 1.72, q4(0) = 0.005, p1(0) = 0., p2(0) = 0., p3(0) = 0., p4(0) = 0., q1(0) = 0., q2(0) = 0., q5(0) = 0., q6(0) = 0., p6(0) = 1.042500000\}$$

```
> sol:=dsolve(eqdp union eqdq union
  eq0,{p1(t),p2(t),p3(t),p4(t),p5(t),p6(t),

  q1(t),q2(t),q3(t),q4(t),q5(t),q6(t)},type=numeric,relerr=1e-5,ab
  serr=1e-5,stiff=true,output=array([seq(0.1*i,i=0..500)]),
  maxfun=30000):

> q5d:=evalf(subs({q4(t)=1.5,p5(t)=0,p6(t)=1},eval(diff(q5(t),t),e
  qdq)));
```

$$q5d := -0.7987970008$$

```
> P1[0]:=subs(eq0,p1(0));P2[0]:=subs(eq0,p2(0));P3[0]:=subs(eq0,p3
  (0));P4[0]:=subs(eq0,p4(0));P5[0]:=subs(eq0,p5(0));P6[0]:=subs(e
  q0,p6(0));
```

$$P1_0 := 0$$

$$P2_0 := 0$$

$$P3_0 := 0$$

$$P4_0 := 0$$

$$P5_0 := 1.042489639$$

$$P6_0 := 1.042500000$$

```
> Q1[0]:=subs(eq0,q1(0));Q2[0]:=subs(eq0,q2(0));Q3[0]:=subs(eq0,q3
  (0));Q4[0]:=subs(eq0,q4(0));Q5[0]:=subs(eq0,q5(0));Q6[0]:=subs(e
  q0,q6(0));
```

$$Q1_0 := 0$$

$$Q2_0 := 0$$

$$Q3_0 := 1.72$$

$$Q4_0 := 0.005$$

$$Q5_0 := 0$$

$$Q6_0 := 0$$

```
> eqPQ:={p1(t)=P1[i-1],p2(t)=P2[i-1],p3(t)=P3[i-1],p4(t)=P4[i-1],p
  5(t)=P5[i-1],p6(t)=P6[i-1],

  q1(t)=Q1[i-1],q2(t)=Q2[i-1],q3(t)=Q3[i-1],q4(t)=Q4[i-1],q5(t)=Q5
  [i-1],q6(t)=Q6[i-1]};
```

$$eqPQ := \{p3 = P3_{i-1}, p4 = P4_{i-1}, p5 = P5_{i-1}, p6 = P6_{i-1}, q1 = Q1_{i-1}, q2 = Q2_{i-1}, q3 = Q3_{i-1}, q4 = Q4_{i-1}, q5 = Q5_{i-1}, q6 = Q6_{i-1}, p1 = P1_{i-1}, p2 = P2_{i-1}\}$$

```
> eqPQm:={p1(t)=P1m,p2(t)=P2m,p3(t)=P3m,p4(t)=P4m,p5(t)=P5m,p6(t)=
  P6m,

  q1(t)=Q1m,q2(t)=Q2m,q3(t)=Q3m,q4(t)=Q4m,q5(t)=Q5m,q6(t)=Q6m};
```

$$eqPQm := \{q1 = Q1m, q2 = Q2m, q3 = Q3m, q4 = Q4m, q5 = Q5m, q6 = Q6m, p1 = P1m, p2 = P2m, p3 = P3m, p4 = P4m, p5 = P5m, p6 = P6m\}$$

```
> i:=1;evalf(subs(eqPQ,eval(diff(q5(t),t),eqdq)));
```

$$i := 1$$

$$1.200010000$$

```
> T[0]:=0:dt:=0.0025;for i from 1 to 1 do
    T[i]:=T[i-1]+dt*0.060;
    P1[i]:=P1[i-1]+dt*evalf(subs(eqPQ,eval(diff(p1(t),t),eqdp))):
    P2[i]:=P2[i-1]+dt*evalf(subs(eqPQ,eval(diff(p2(t),t),eqdp))):
    P3[i]:=P3[i-1]+dt*evalf(subs(eqPQ,eval(diff(p3(t),t),eqdp))):
    P4[i]:=P4[i-1]+dt*evalf(subs(eqPQ,eval(diff(p4(t),t),eqdp))):
    P5[i]:=P5[i-1]+dt*evalf(subs(eqPQ,eval(diff(p5(t),t),eqdp))):
    P6[i]:=P6[i-1]+dt*evalf(subs(eqPQ,eval(diff(p6(t),t),eqdp))):
    Q1[i]:=Q1[i-1]+dt*evalf(subs(eqPQ,eval(diff(q1(t),t),eqdq))):
```

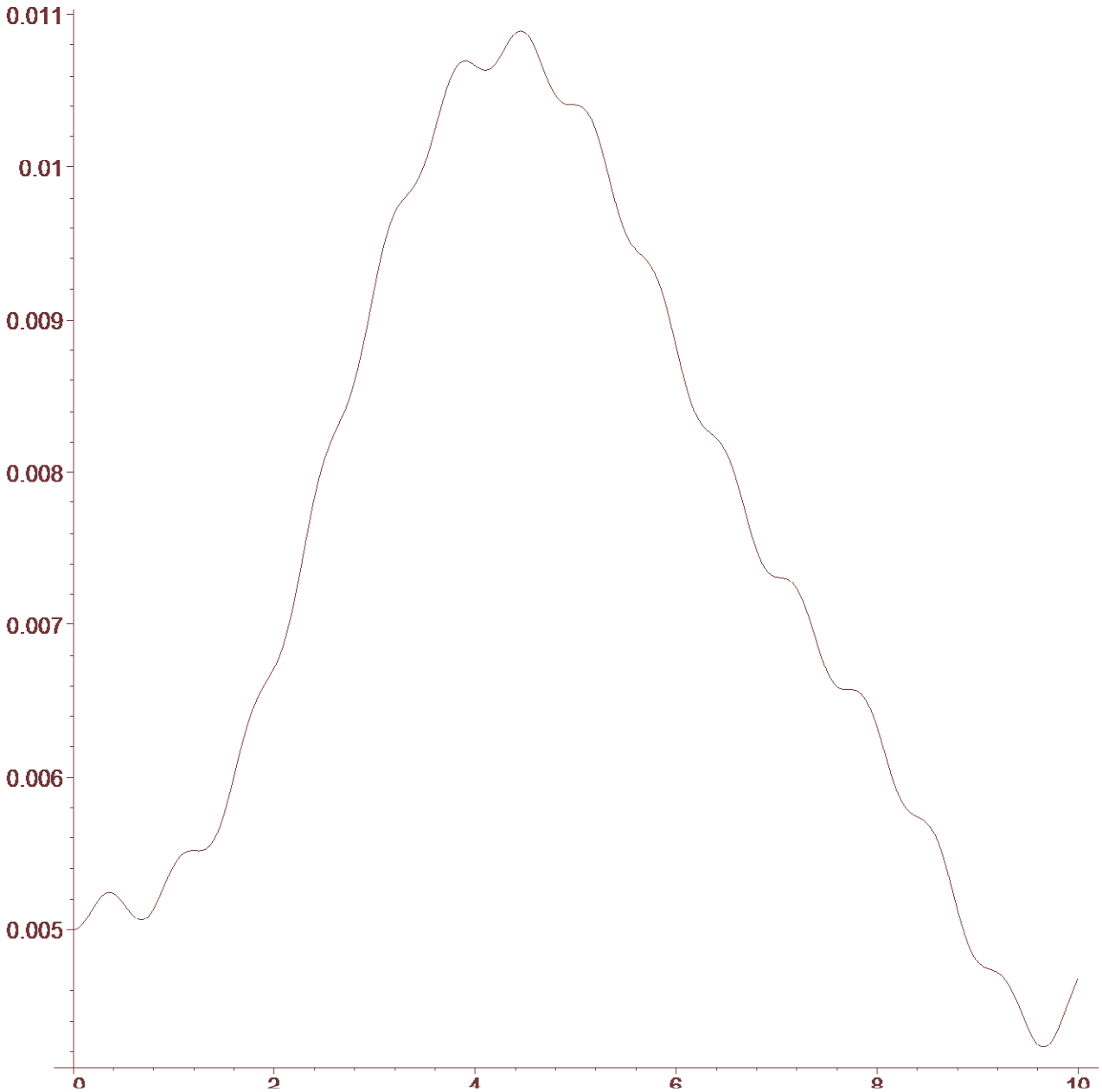
```
  Q2[i]:=Q2[i-1]+dt*evalf(subs(eqPQ,eval(diff(q2(t),t),eqdq))):
  Q3[i]:=Q3[i-1]+dt*evalf(subs(eqPQ,eval(diff(q3(t),t),eqdq))):
  Q4[i]:=Q4[i-1]+dt*evalf(subs(eqPQ,eval(diff(q4(t),t),eqdq))):
  Q5[i]:=Q5[i-1]+dt*evalf(subs(eqPQ,eval(diff(q5(t),t),eqdq))):
  Q6[i]:=Q6[i-1]+dt*evalf(subs(eqPQ,eval(diff(q6(t),t),eqdq))):
end do:
```

$$dt := 0.0025$$

```
> T[0]:=0:dt:=0.01;for i from 1 to 1000 do
  T[i]:=T[i-1]+dt;
  P1m:=P1[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(p1(t),t),eqdp))):
  P2m:=P2[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(p2(t),t),eqdp))):
  P3m:=P3[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(p3(t),t),eqdp))):
  P4m:=P4[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(p4(t),t),eqdp))):
  P5m:=P5[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(p5(t),t),eqdp))):
  P6m:=P6[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(p6(t),t),eqdp))):
  Q1m:=Q1[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(q1(t),t),eqdq))):
  Q2m:=Q2[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(q2(t),t),eqdq))):
  Q3m:=Q3[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(q3(t),t),eqdq))):
  Q4m:=Q4[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(q4(t),t),eqdq))):
  Q5m:=Q5[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(q5(t),t),eqdq))):
  Q6m:=Q6[i-1]+dt/2*evalf(subs(eqPQ,eval(diff(q6(t),t),eqdq))):

  P1[i]:=P1[i-1]+dt*evalf(subs(eqPQm,eval(diff(p1(t),t),eqdp))):

  P2[i]:=P2[i-1]+dt*evalf(subs(eqPQm,eval(diff(p2(t),t),eqdp))):

  P3[i]:=P3[i-1]+dt*evalf(subs(eqPQm,eval(diff(p3(t),t),eqdp))):

  P4[i]:=P4[i-1]+dt*evalf(subs(eqPQm,eval(diff(p4(t),t),eqdp))):

  P5[i]:=P5[i-1]+dt*evalf(subs(eqPQm,eval(diff(p5(t),t),eqdp))):

  P6[i]:=P6[i-1]+dt*evalf(subs(eqPQm,eval(diff(p6(t),t),eqdp))):

  Q1[i]:=Q1[i-1]+dt*evalf(subs(eqPQm,eval(diff(q1(t),t),eqdq))):

  Q2[i]:=Q2[i-1]+dt*evalf(subs(eqPQm,eval(diff(q2(t),t),eqdq))):

  Q3[i]:=Q3[i-1]+dt*evalf(subs(eqPQm,eval(diff(q3(t),t),eqdq))):

  Q4[i]:=Q4[i-1]+dt*evalf(subs(eqPQm,eval(diff(q4(t),t),eqdq))):

  Q5[i]:=Q5[i-1]+dt*evalf(subs(eqPQm,eval(diff(q5(t),t),eqdq))):

  Q6[i]:=Q6[i-1]+dt*evalf(subs(eqPQm,eval(diff(q6(t),t),eqdq))):
  end do:
```

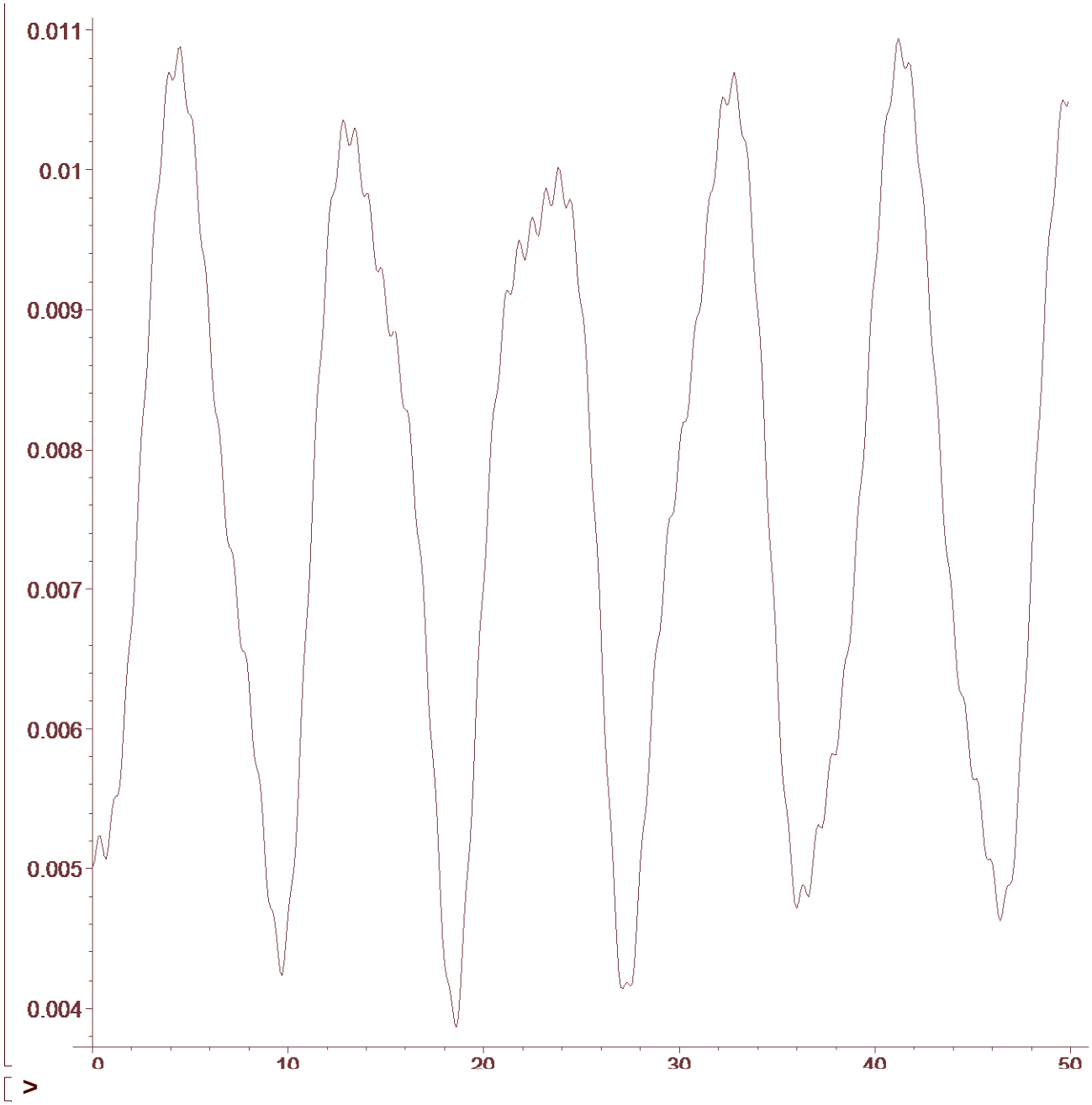
$$dt := 0.01$$

```
>
> with(plots):
Warning, the name changecoords has been redefined
> listplot([seq([T[i],Q4[i]],i=1..1000)]);
```




```
> eval(sol[1,1]);listplot([eval(seq([sol[2,1][i,1],sol[2,1][i,11]]
  ,i=1..500))]);
```

$$[t, p1, p2, p3, p4, p5, p6, q1, q2, q3, q4, q5, q6]$$


0.011
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0
10
20
30
40
50
v